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A fetching and descriptive title for your manuscript

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Abstract. Insert your abstract here. The abstract should be a short paragraph that details what the current paper sets out to do, and a high-level description of how it goes about what it does. Also, you should avoid putting citations, like this one [3], in the abstract.

1 Introduction

Ideally, the introduction should introduce the background of the problem at hand and motivate why mathematicians (and/or others) are interested in studying it. This should be aimed at the level of undergraduates who are not experts in your area. We'd also like the introduction to contain a road map for the article; i.e., a description of what to expect in each section.

2 The title of the second section

The class file contains definitions for several environments (e.g., `thm`, `corollary`, `lemma`, etc). Examples follow below.

Important definitions should be set in the `defin` environment.

Definition 2.1. Let $p \in \mathbf{N}$ and suppose the following hold:

P1. If $d \in \mathbf{N}$ such that $d \mid p$, then $d = 1$ or $d = p$.

P2. $p \neq 1$.

Then we say that p is a *prime number*.

[*]Thanks to all those hapless Pythagoreans.
[†]Thanks for nothing, Bernoulli.
[‡]You should really be thanking me.

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Use the `label` command for reference to these environments later on in your document. The class file uses the `cleveref` package which determines the type of label being referenced:

	LaTeX code	Result
Using `ref`:	`\textbf{definition \ref{prim-def}}`	**definition 2.1**
Using `cref`:	`\cref{prim-def}`	**definition 2.1**

Lemma 2.2. *If $n \in \mathbf{N}$, then there exists a prime number p such that $p \mid n$.*

Proof. Suppose, for contradiction, that there exists $n \in \mathbb{N}$ such that $n > 1$ and n is not divisible by any prime. Let

$$S = \{n \in \mathbf{N} : n > 1,\ n \text{ is not divisible by a prime}\}.$$

Then S is a non-empty subset of $\mathbf{N}$. The well-ordering principle[1] fashions a least element of S, say m. Note that m is not prime (otherwise it would be divisible by a prime, namely itself). Since m is not prime, there exists $a \in \mathbf{N}$ such that $a \mid m$ and $a \neq 1$ and $a \neq m$. Let $b \in \mathbf{Z}$ such that $ab = m$. Note that $b \in \mathbf{N}$ since $a, m \in \mathbf{N}$. What's more, $b > 1$ since otherwise $a = m$. So it must be that $1 < a, b < m$. Since divisibility is transitive, it follows that neither a nor b is divisible by a prime. But then $a \in S$ (and so is b) and $a = m/b < m$ contradicting the minimality of m. Thus it must be that S is empty which proves the lemma. □

Theorem 2.3 (Euclid)**.** *There are infinitely many prime numbers.*

Proof. Let $p_1, p_2, \ldots, p_k$ be primes, and let

$$n = p_1 p_2 \cdots p_k + 1.$$

By **lemma 2.2**, there exists a prime p such that $p \mid n$. Suppose, for contradiction, that $p = p_i$ for some $i = 1, 2, \ldots, k$. It follows that $p \mid n - p_1 p_2 \cdots p_k = 1$, but this is absurd. So it must be that $p \neq p_i$ for any $i = 1, 2, \ldots k$. This shows that a new prime can be constructed from any finite list of primes. Hence the number of primes is not finite. □

Use the `cite` command to cite references. For example, the above proof can be found in Euclid's Elements [1]. See below for how to add citations to the `thebibliography` environment.

[1]An excellent principle.

3 A title for the third section

There is an `example` environment.

Example 3.1. The geometric series
$$\sum_{k=0}^{\infty} \frac{1}{2^k}$$
converges to 2.

When starting a sentence with a reference, use the `Cref` command. **Example 3.1** can be used to prove that 2 is not the only prime.

Theorem 3.2. *The set of primes includes more numbers than just* 2.

Proof. If 2 were the only prime number, then the fundamental theorem of arithmetic would give us that
$$\sum_{n=1}^{\infty} \frac{1}{n} = \sum_{k=0}^{\infty} \frac{1}{2^k}.$$
The series on the left diverges whereas the series on the right converges. This is impossible, so there must be more prime numbers beyond the number 2. □

There is a `remark` environment as well.

Remark 3.3. The idea in **theorem 3.2** can be extended to show that the set of primes includes more numbers than just 2 and 3. In fact, Euler went on to prove that if the number of primes were finite, then the harmonic series would converge.

4 A title for the fourth section

There are `prop` and `corollary` environments as well.

Proposition 4.1. *Every natural number can be written uniquely as a product of a square-free number and a square.*

As a corollary to **proposition 4.1** we obtain

Corollary 4.2. *Let* $\pi(n)$ *denote the number of primes less than or equal to* n*. Then*
$$\pi(n) \geq \frac{\log n}{2\log 2}.$$

Proof. There are no more than $2^{\pi(n)}$ square free numbers less than n. Also, there are no more than $\sqrt{n}$ squares less than n. It follows from **proposition 4.1** that
$$n \leq 2^{\pi(n)}\sqrt{n}.$$
The corollary follows by applying log. □

For an unnumbered remark, use the `xrem` environment.

Remark. Note that **corollary 4.2** implies that there are infinitely many primes.

References

[1] Euclid, *Euclid's Elements*, the Thomas L. Heath translation, Green Lion Press, Santa Fe, NM, 2002. MR1932864

[2] Euler, *Foundations of differential calculus*, translated from the Latin by John D. Blanton, Springer-Verlag, New York, 2000. MR1753095

[3] P. G. L. Dirichlet, Gedächtnißrede auf Carl Gustav Jacob Jacobi, in *Nachrufe auf Berliner Mathematiker des 19. Jahrhunderts*, 6–34, Teubner-Arch. Math., 10, Teubner, Leipzig. MR1104895

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